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Scattering phase function for fibrous media

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Abstract—The formulation and evaluation of the effective phase function for fibrous media with any fiber orientation are presented based on the assumption that the fibers are infinite cylinders and scattering is in the independent regime. The phase function is derived in terms of the scattering angle which is the angle between the incident and the scattered radiation. Unlike spherical particles which scatter radiation into all directions, radiation scattered by cylindrical fibers only propagates into limited angular ranges. Hence, the maximum value of the scattering angle for radiation scattered by a collection of fibers depends on the orientation of the fibers relative to the direction of the incident radiation. For a given fiber orientation both the angular variation and the maximum scattering angle of the phase function are different for different incident directions. If the fibers are randomly oriented in space, the angular variation of the phase function is independent of the incident direction and the maximum scattering angle is π. The phase function for fibrous media always exhibits a strong peak in the direction of incident radiation, indicating that the scattering is highly anisotropic.

INTRODUCTION

THE INTERACTION of radiation with cylindrical particles plays an important role in many physical phenomena, such as in the transmission of radiation through the atmosphere and in the radiative heat transfer through fibrous insulation [1–8]. In these cases the wavelength of radiation is of the same order of magnitude as the diameter of the cylinders the lengths of which are much larger than the diameter. It has been shown that the radiation properties of finite and infinitely long cylinders becomes almost identical if the length of the cylinder is much longer than its diameter [9, 10].

The atmospheric radiation balance and the atmospheric visibility are known to be affected by ice clouds and cirrus. By modeling the non-spherical ice crystals as infinite cylinders, the scattering phase function and the de-polarization factor have been investigated by Liou [1]. The ice cylinders were assumed to be oriented in planes. The phase function was shown to vary with the direction of the incident radiation.

For studies on thermal radiation through fibrous media, fibers were also modeled as infinite cylinders [2–8]. Fibers in many insulation materials are woven in specific directions. Houston and Korpela [2] considered the case of fibers randomly oriented in space. The phase function of this type of fibrous medium is independent of the incident direction. Other studies [3–5] have generally neglected the effect of fiber orientation on the phase function and on the radiation heat transfer analyses. However, the extinction and scattering coefficients of a fibrous medium were shown to be strongly influenced by the orientations of both the fiber and the incident radiation [6]. By accounting for the fiber orientation and the geometry for the scattered radiation, the single fiber phase function was employed to develop radiation heat transfer models, based on the two-flux approximation, applicable to fibrous media with any fiber orientation [7, 8].

Although the influence of fiber orientation on radiation heat transfer has been investigated within the context of the two-flux model, the accuracy of two-flux models is limited by the adequacy of the assumption of semi-isotropic distribution for the scattered radiation. Accurate analysis requires the use of either Monte-Carlo or discrete ordinate methods. These approaches require detailed knowledge of the phase function. However, no analysis has yet been presented on the phase function for fibrous media which accounts for the orientation of the fibers.

The scattering phase function for fibrous media requires special consideration due to the two-dimensional nature of the scattered radiation by cylindrical fibers. Unlike spherical particles which scatter radiation into all directions in space, radiation scattered by cylindrical fibers is confined to propagate along a conic surface [9]. The apex angle of this cone is dictated by the direction of the incident radiation relative to the fiber axis. Hence, the angular range of the scattered radiation by a collection of fibers is strongly influenced by both the fiber orientation and the incident direction. On the contrary the uni-dimensional geometry of spherical particles precludes the need to consider the effect of orientation on the scattering of radiation.

This paper investigates the influence of fiber orientation on the phase function of fibrous media. The fibers are modeled as infinite cylinders and the scattering of radiation is assumed to be in the independent regime. The phase function will be evaluated for different combinations of the incident direction and fiber orientation. In the following sections, the phase function for a single cylinder is first discussed. This

NOMENCLATURE

C_s	scattering cross section
d^2F	fiber orientation distribution function
$i(\theta, \phi)$	intensity function
j	$\sqrt{-1}$
k	imaginary part of m
m	complex index of refraction, $n - jk$
M	total number of weave directions
n	real part of m
$N(r)\,dr$	fiber number size distribution
p	phase function for single cylinder
P	distribution of the scattered radiation
P_s	phase function for fibrous media
r	radius of cylinder
R	unit vector
x_i	fraction of fibers oriented in the ith polar direction.

Greek symbols

η	angle between incident and scattering radiation
θ	angle of observation
λ	wavelength
ξ	polar angle
σ_s	scattering coefficient
ϕ	angle of incidence, $\lvert\pi/2 - \phi_c\rvert$
ϕ_c	half apex cone angle
φ	azimuthal angle on the plane normal to the incident direction
ω	azimuthal angle
Ω	solid angle.

Subscripts

f	fiber
i	$1, 2, \ldots, M$
s	scattered radiation.

is then followed by the consideration of the phase function for a medium of fibers.

THEORETICAL CONSIDERATION

The scattering of radiation by infinite cylinders differs significantly from that by finite size particles. For finite particles such as spheres, the scattered radiation propagates out as spherical waves spanning the 4π steradian solid angle [9]. The scattered radiation can be observed by placing a detector at any angular position in space.

The scattering of radiation by finite cylinders is quite similar to that by spheres [9, 10]. However, as the aspect ratio (length/diameter) of the cylinder increases, the scattered waves become increasingly confined to a surface. When the aspect ratio approaches 100, the scattered radiation propagates only along the surface of a cone similar to that for an infinite cylinder [10]. The half apex angle of the cone is the angle between the incident direction and the cylinder axis. The scattered radiation can only be detected if the detector is inclined from the fiber axis at an angle equal to the half apex angle of the cone.

Single fiber phase function

Figure 1 depicts the scattering of radiation by an infinite cylinder at oblique incidence relative to the fiber. The half apex angle of the cone is ϕ_c and the angle of incidence is $\phi = |\pi/2 - \phi_c|$. The azimuthal angle of the scattered radiation relative to the incident direction on a plane normal to the fiber axis is denoted by θ, the angle of observation. The scattering cross section per unit length of the infinite cylinder is given by [9]

$$C_s(\phi) = \frac{\lambda}{\pi^2} \int_0^{2\pi} i(\theta, \phi)\, d\theta \quad (1)$$

where λ is the wavelength and $i(\theta, \phi)$ the intensity function which depends on both the fiber diameter and the index of refraction. The intensity function is derived from the solution of Maxwell's equations for the interaction of electromagnetic radiation with infinite cylinders [9]. The implicit dependence on the refractive index $m = n - jk$, $j = \sqrt{-1}$ has been omitted for brevity. The scattering coefficient is obtained by integrating C_s over the size distribution $N(r)\,dr$ as

$$\sigma_s = \int_{r_1}^{r_2} C_s N(r)\, dr \quad (2)$$

which varies with the incidence angle ϕ.

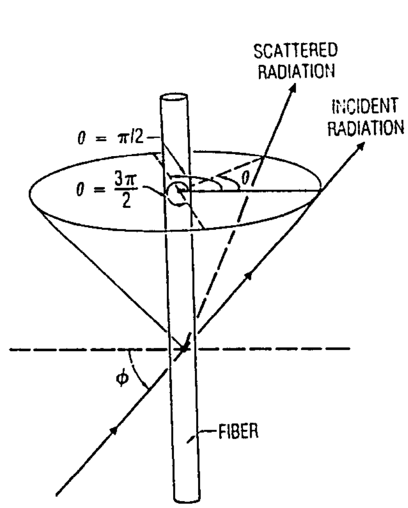

FIG. 1. Scattering of radiation by a single cylinder.

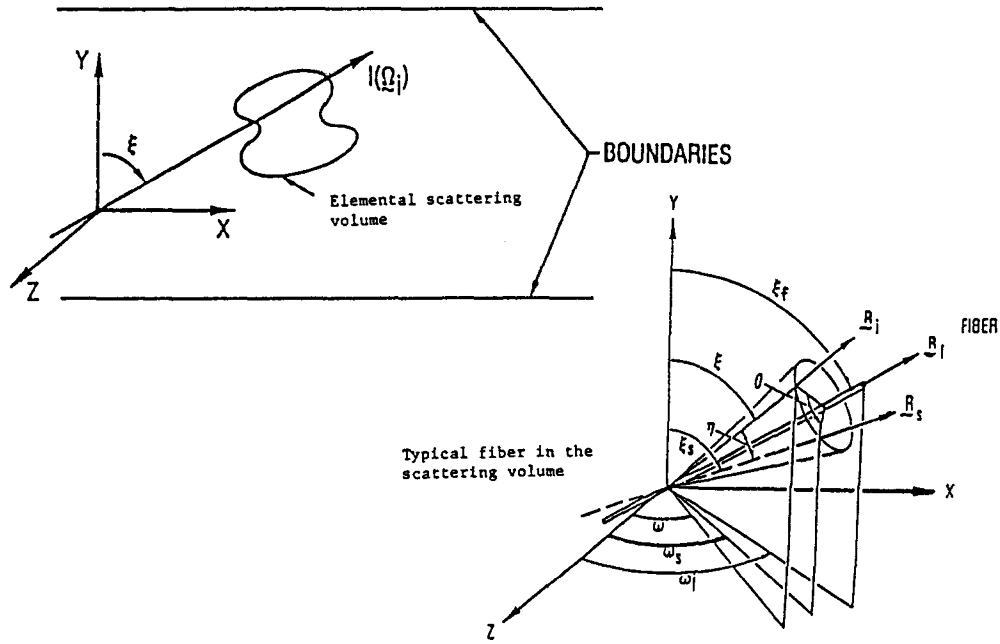

FIG. 2. Orientation of a typical fiber in an elemental volume.

The phase function for a single fiber describes the distribution of the scattered radiation relative to the fiber coordinates. The amount of scattered energy is proportional to the scattering cross section of the cylinder at the incidence angle ϕ. The single fiber phase function can be defined as

$$p(\theta,\phi) = i(\theta,\phi)\bigg/\int_0^{2\pi} i(\theta,\phi)\,\mathrm{d}\theta \tag{3a}$$

$$= \frac{\lambda}{\pi^2}\frac{i(\theta,\phi)}{C_s} \tag{3b}$$

which becomes unity when integrated over $\mathrm{d}\theta$ from 0 to 2π.

Although the phase function for a single fiber is quite simple and easy to compute, this definition is difficult to apply to radiative transfer calculations for fibrous media in which fibers can be oriented in any direction. It is obvious that for radiation traversing a collection of fibers, the same value of θ generally refers to different directions in space for different orientations of the fibers. The sense of direction of θ is lost when describing the scattered radiation by a medium of fibers. Hence, the phase function for a fibrous medium must be described in terms of a global coordinate system in which the orientations of the fibers are specified.

Phase function for a medium of fibers

The phase function for a fibrous medium can be defined by first considering a fiber oriented with respect to a global coordinate system XYZ as depicted in Fig. 2. The phase function relative to this global coordinate system is given by [6]

$$p(\eta,\phi_c) = \frac{4\lambda}{\pi^2}\frac{i(\eta,\phi_c)}{C_s(\phi_c)\sin\theta\sin^2\phi_c} \tag{4}$$

where the scattering angle η is the included angle between the incident and the scattered radiation. The half apex angle of the cone of scattered radiation (ϕ_c) is related to the incident and scattered directions by

$$\cos\phi_c = \sin\xi * \sin\xi_f * \cos(\omega-\omega_f) + \cos\xi * \cos\xi_f \tag{5a}$$

$$= \sin\xi_s * \sin\xi_f * \cos(\omega_s-\omega_f) + \cos\xi_s * \cos\xi_f \tag{5b}$$

where ξ and ω are the polar and azimuthal angles, and the subscripts s and f refer to the scattered radiation and the fiber, respectively. The expression for η can be obtained by replacing ϕ_c, ξ_f, and ω_f in equation (5a) by η, ξ_s, and ω_s, where (ξ_s,ω_s) refers to the direction of the scattered radiation. In addition the transformation given by

$$\cos\theta = \frac{\cos\eta-\cos^2\phi_c}{\sin^2\phi_c} \tag{6}$$

defines the relationship between the scattering angle η and the angles θ and ϕ_c which are specified relative to a fiber. The effective phase function for a fibrous medium is strongly influenced by the scattering characteristics of each fiber. It is obtained by considering the distribution of radiation scattered by all the fibers normalized by the effective scattering cross section of the fibers.

The distribution of the radiation scattered by an elemental volume of a fibrous medium is obtained by integrating the product $C_s p$ over the size distribution and orientations of the fibers. By combining C_s with p in equation (4), the distribution of the scattered radiation due to radiation traversing the medium in the direction (ξ, ω) is given by

$$P(\xi,\omega;\eta) = \int_{r_1}^{r_2}\int_{\omega_f}\int_{\xi_f} C_s p\,\mathrm{d}^2F\,N(r)\,\mathrm{d}r \quad (7)$$

where d^2F specifies the orientation of the fibers. This function P is equivalent to the product of the scattering coefficient and the phase function in the conventional sense. It is unique for each incident direction.

The effective scattering coefficient for a fibrous medium can be obtained by integrating the scattered radiation over all directions. By using the distribution function of the scattered intensity given by equation (7), it becomes

$$\sigma_s(\xi,\omega) = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} P\sin\eta\,\mathrm{d}\eta\,\mathrm{d}\varphi = \frac{1}{2}\int_0^{\pi} P\sin\eta\,\mathrm{d}\eta \quad (8)$$

where η and φ are regarded as the spherical polar angles of the scattered radiation relative to the incident direction, and φ the azimuthal angle on the plane normal to the incident direction.

The effective scattering coefficients can be obtained alternatively from the single fiber scattering cross section defined by equation (2). Hence, it can be evaluated as

$$\sigma_s(\xi,\omega) = \int_{r_1}^{r_2}\int_{\omega_f}\int_{\xi_f} C_s\,\mathrm{d}^2F\,N(r)\,\mathrm{d}r. \quad (9)$$

Comparison of σ_s evaluated from equations (8) and (9) provides the consistency check for the accuracy of the numerical values of P.

The scattering phase function in the conventional sense is obtained by normalizing P with $\sigma_s(\xi,\omega)$ as

$$P_s(\xi,\omega;\eta) = P/\sigma_s(\xi,\omega). \quad (10)$$

Integrating P_s over the scattering solid angle ($\mathrm{d}\Omega = \sin\eta\,\mathrm{d}\eta\,\mathrm{d}\varphi$) yields 4π. It is re-iterated that P_s is derived based on the assumption of independent scattering. P_s is the effective phase function for a given distribution of fiber orientation and is independent of the porosity, i.e. volume fraction, of the fibrous medium.

Phase function for specific fiber orientations

A closed form expression cannot be obtained for P_s due to its complicated functional dependence. Numerical evaluation of P_s is more convenient and efficient. In this paper specific attention is devoted to fibers oriented in discrete polar angles and randomly oriented in the azimuthal direction. For this type of fiber orientation, the orientation distribution function is

$$\mathrm{d}^2F = x_i\delta(\xi_f-\xi_{fi})\,\mathrm{d}\xi_f\,\mathrm{d}(\omega_f-\omega)/\pi \quad (11)$$

where δ is the Kronecker delta function, x_i the fraction of fibers oriented in the polar direction ξ_{fi}. Due to symmetry the range of $\omega_f-\omega$ is taken from 0 to π and the normalization factor is π. By applying the coordinate transformations of equations (5) and (6), the phase function becomes

$$P_s(\xi,\omega;\eta) = \frac{4\lambda}{\pi^3\sigma_s(\xi,\omega)} \times \sum_{i=1}^{M} x_i \int_{r_1}^{r_2}\int_{\cos\phi_{c1}}^{\cos\phi_{c2}}\int_0^{\pi/2} G\,\mathrm{d}\xi_f\,\mathrm{d}(\cos\phi_c')N(r)\,\mathrm{d}r \quad (12)$$

where

$$G = \frac{i(\eta,\phi_c')\delta(\xi_f-\xi_{fi})/[(1-\cos\eta)(\cos\eta-2\cos^2\phi_c'+1)]^{1/2}}{[(\cos\phi_{c2}-\cos\phi_c')(\cos\phi_c'-\cos\phi_{c1})]^{1/2}}$$

and σ_s is calculated by using equation (11) in equation (9). The limits of integration are obtained by using equation (5a)

$$\cos\phi_{c1} = \cos\xi\cos\xi_f - \sin\xi\sin\xi_f \quad (13a)$$

$$\cos\phi_{c2} = \cos\xi\cos\xi_f + \sin\xi\sin\xi_f \quad (13b)$$

which correspond to $|\omega-\omega_f| = \pi$ and 0, respectively.

The analytical treatment for fibers with specific azimuthal orientations is similar, except that a delta function indicating the azimuthal fiber directions also needs to be included in the distribution function of equation (11). In reality fibrous media such as fabrics may have multiple weave directions. The phase function would then be evaluated as the summation of the individual P_s corresponding to each combination of ξ_{fi} and ω_{fi} for all the weave directions. Hence, the phase function given by equation (12) is the basic formula from which the phase function for fibrous media with more complicated weave patterns can be derived. The limits of integration for each P_s are again derived from equation (6a).

For the special case of fibers randomly oriented in space, d^2F is given by $\cos\phi\,\mathrm{d}\phi$. The phase function then becomes [2, 6]

$$P_s = \int_{r_1}^{r_2}\int_0^{\pi/2} C_s p\cos\phi\,\mathrm{d}\phi\,N(r)\,\mathrm{d}r \Big/ \int_{r_1}^{r_2}\int_0^{\pi/2} C_s(\phi)\cos\phi\,\mathrm{d}\phi\,N(r)\,\mathrm{d}r$$

$$= \int_{r_1}^{r_2}\int_0^{1} C_s p\,\mathrm{d}(\cos\phi_c')\,N(r)\,\mathrm{d}r \Big/ \int_{r_1}^{r_2}\int_0^{1} C_s(\phi_c')\,\mathrm{d}(\cos\phi_c')\,N(r)\,\mathrm{d}r \quad (14)$$

which is independent of the incident angle.

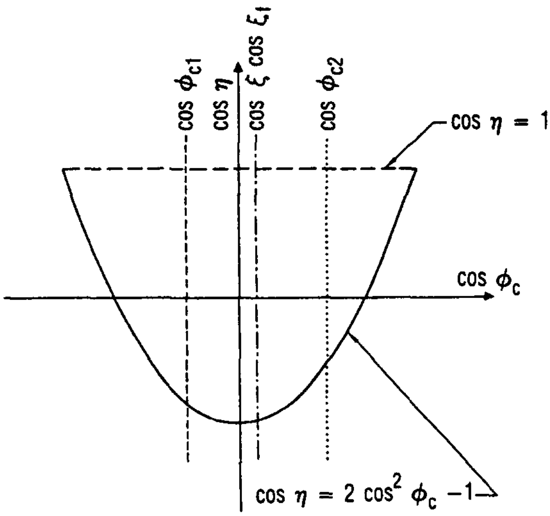

FIG. 3. Variation of the scattering angle with the half apex cone angle.

Due to the two-dimensional nature of the scattered radiation by infinite cylinders, care must be exercised in integrating equations (12) and (14). In particular, the geometry of the cone of scattered radiation imposes strict constraints on the limits of integration for each scattering angle. The effect of these constraints on the limits of integration is discussed below.

Limits of integration and the maximum value of the scattering angle

Because radiation scattered by infinite cylinders propagates along the surface of a cone, the range of the scattering angle η is fixed for each fiber as depicted in Fig. 3. The valid scattering region is enclosed by the line $\cos\eta = 1$ and the parabola $\cos\eta = 2\cos^2\phi_c - 1$. These boundaries are obtained by setting θ equal to 0 and 2π in equation (5). Hence, for a given half apex angle ϕ_c, the scattering angle η cannot exceed η_0 defined by the intersection of the line of constant $\cos\phi_c$ with the parabola. The limits of integration in equations (12) and (14) correspond to two vertical lines of constant $\cos\phi_c$ in Fig. 3. They are not necessarily symmetrical about the line $\cos\phi_c = \cos\xi\cos\xi_f$ which corresponds to $|\omega_f - \omega_i| = \pi/2$ as shown in the figure. For the special case of fibers randomly oriented in space, the upper limit of integration is defined by the locus of the parabola.

For a particular incident direction and fiber orientation, the maximum value of the scattering angle η_{max} is obtained from equations (5) and (6) as

$$\cos\eta_{max} = \text{minimum of} \begin{cases} 2\cos^2\phi_{c1} - 1 \\ 2(\cos\xi\cos\xi_f)^2 - 1 \\ 2\cos^2\phi_{c2} - 1 \end{cases} \quad (15)$$

which correspond to $|\omega - \omega_f| = \pi$, $\pi/2$, and 0, respectively. For the range of $\eta < \eta_0$, the limits of integration follow the locus of the parabola. Physically, this means that only those fibers forming half apex cone angles smaller than those defined by the boundary of the parabola can scatter radiation into the angle η. The reduced range of integration denotes that a limited number of fibers contributes to the scattered radiation at that angle η.

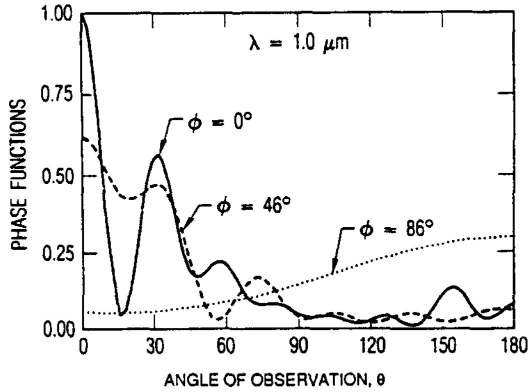

FIG. 4. Scattering phase function for a single cylinder at $\lambda = 1.0$ μm.

RESULTS AND DISCUSSION

For the purpose of illustration, the phase functions given by equations (12) and (14) were evaluated for glass fibers of 1.0 μm radius and at wavelengths of 1.0 and 9.18 μm. The optical constants of glass are $m = 1.507$ (non-absorbing) at $\lambda = 1.0$ μm and $m = 1.05 - 1.08j$ (absorbing) at $\lambda = 9.18$ μm [11].

Figures 4 and 5 show the single fiber phase functions at various incident angles for $\lambda = 1.0$ and 9.18 μm, respectively. It is re-iterated that the incident and scattering directions are defined with respect to the fiber coordinates θ and ϕ as shown in Fig. 1. The scattering of radiation is symmetrical about the plane containing the fiber axis and the incident direction. For non-absorbing fibers (Fig. 4) the phase function shows

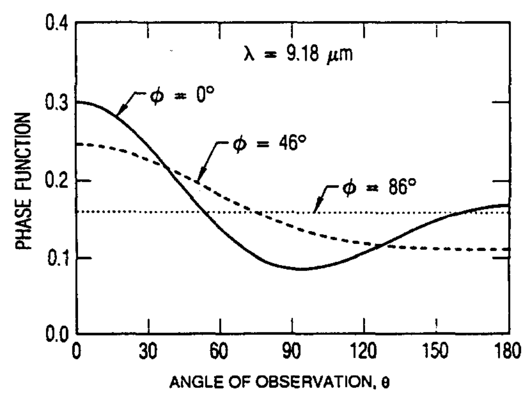

FIG. 5. Scattering phase function for a single cylinder at $\lambda = 9.18$ μm.

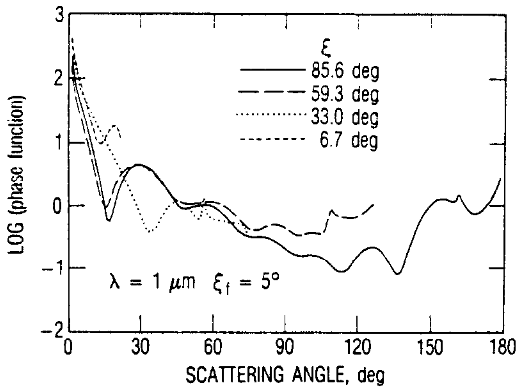

FIG. 6. Scattering phase function for a fibrous medium with fibers included at 5° from the normal to the planar boundaries ($\lambda = 1.0\ \mu$m).

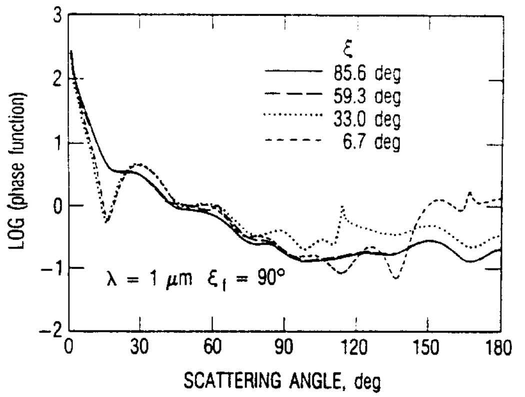

FIG. 8. Scattering phase function for a fibrous medium with fibers oriented parallel to the planar boundaries ($\lambda = 1.0$ μm).

more pronounced angular variation than that for absorbing fibers (Fig. 5).

The phase functions for fibrous media are shown in Figs. 6–11. Fibers in the medium are assumed to be all inclined at 5.0°, 45.0°, and 90° from the $+Z$ axis and are randomly oriented in the azimuthal directions. For a fibrous medium with multiple discrete polar fiber directions, the effective phase function is simply the weighted average of that for fibers oriented in each direction. The polar angles of the incident radiation are 6.7°, 33.0°, 59.3°, and 85.6°. In addition, the case of fibers randomly oriented in space is also considered. It is re-iterated that in the consideration of a medium of fibers, all angles are defined with respect to the global coordinate system which is different from that for any individual fiber.

The phase functions for radiation at $\lambda = 1.0\ \mu$m incident at different polar angles are shown in Figs. 6–8. The glass fibers are non-absorbing at this wavelength because the index of refraction is real ($m = 1.57$). In Fig. 6 the fibers are inclined at 5.0° from the normal to the boundaries ($+Z$ axis) and the maximum value of the scattering angle η_{max} is different for different incident angles. For the incident radiation at 6.7°, η_{max} is 23.4°. Therefore, all radiation is scattered into the upper half space, i.e. $0 \leqslant \xi \leqslant \pi/2$. The angle η_{max} increases with larger incidence angles because the maximum half apex cone angle for individual fibers becomes larger. On the other hand, η_{max} also increases with higher polar orientation of the fibers as shown in Fig. 7 for $\xi_f = 45.0°$. Figure 8 shows the results for $\xi_f = 90.0°$, i.e. fibers randomly oriented in planes. In this case the axes of fibers at some azimuthal angles are normal to the incident radiation. Therefore, the maximum scattering angle is π regardless of the angle of the incident radiation. The phase function shows a strong peak in the forward direction, indicating that the scattering is highly anisotropic.

Figures 9–11 show the phase functions at $\lambda = 9.18$ μm for fibers oriented at 5.0°, 45.0°, and 90°, respectively. At this wavelength the index of refraction is complex ($m = 1.05 - 1.08j$) and the fibers are absorbing. The maximum scattering angle η_{max} for these phase functions are identical to those at $\lambda = 1.0\ \mu$m. This is, of course, not unexpected because η_{max} is dependent only on the directions of fibers and the incident radiation. The phase functions display considerably less pronounced angular variation than those previous cases because the fibers are absorbing. Smoothing of the angular variation is also evident for

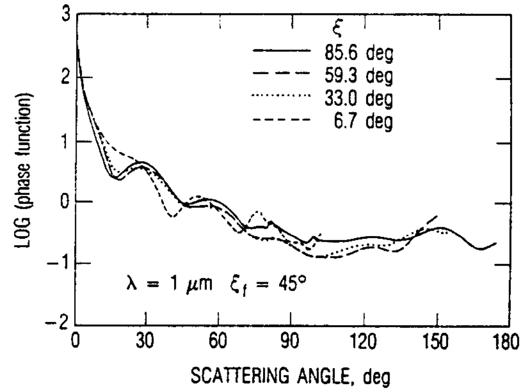

FIG. 7. Scattering phase function for a fibrous medium with fibers inclined at 45° from the normal to the planar boundaries ($\lambda = 1.0\ \mu$m).

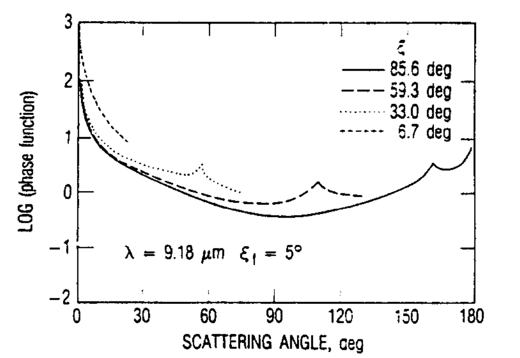

FIG. 9. Scattering phase function for a fibrous medium with fibers inclined at 5° from the normal to the planar boundaries ($\lambda = 9.18\ \mu$m).

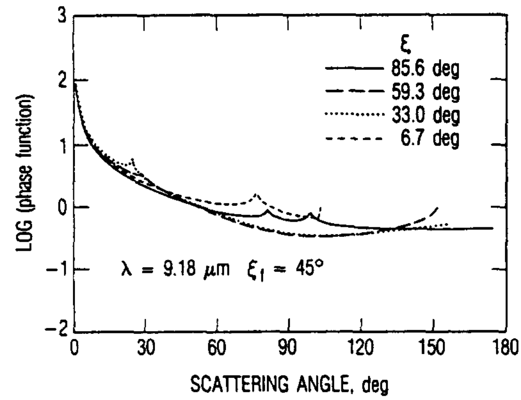

FIG. 10. Scattering phase function for a fibrous medium with fibers inclined at 45° from the normal to the planar boundaries ($\lambda = 9.18$ μm).

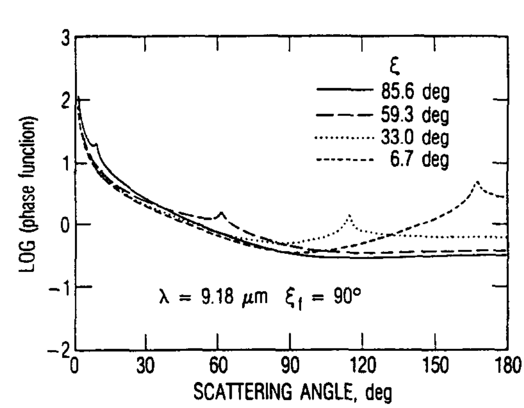

FIG. 11. Scattering phase function for a fibrous medium with fibers oriented parallel to planar boundaries ($\lambda = 9.18$ μm).

a single fiber as shown in Fig. 4. In addition, the phase functions of absorbing fibers are less peaked in the forward direction than those of the non-absorbing fibers.

The phase functions for fibrous media with fibers randomly oriented in space are shown in Fig. 12. As evident from physical intuition, these phase functions are independent of the incident angle and the maximum scattering angle is π. The phase function for absorbing fibers shows much less angular variation and is less peaked than that for non-absorbing fibers.

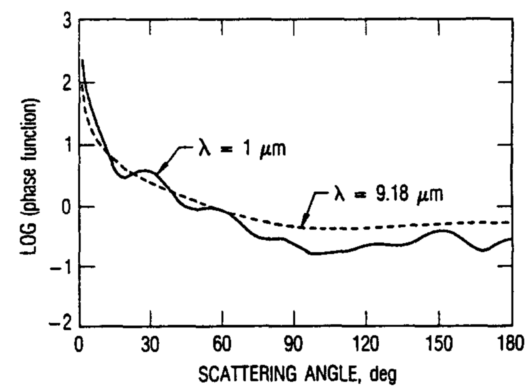

FIG. 12. Scattering phase function for fibrous media with fibers randomly oriented in space for $\lambda = 1.0$ and 9.18 μm.

The above results revealed a significant difference in the phase function between spherical and very long cylindrical particles. For spherical particles the extinction and scattering cross sections are independent of the direction of the incident radiation. The scattered radiation propagates out as spherical waves which span the 4π steradian solid angle. Hence, the phase function depends only on the scattering angle which is the angle between the incident and the scattered radiation. For infinite cylindrical particles, however, the radiative cross sections vary with the incidence angle. The scattered radiation is two-dimensional and is confined to the surface of a cone. As a result the radiative coefficients and the angular variation of the phase function for a medium of fibers are strongly dependent on the orientations of both the fibers and the incident radiation. Only in the special case of fibers randomly oriented in space are the angular variation of the phase function and the radiative coefficients independent of the fiber orientation and the incident direction.

It is reiterated that the phase function for fibrous media has been derived in terms of the scattering angle which is the angle between the incident and the scattered radiation. However, due to the two-dimensional characteristics of radiation scattered by cylindrical fibers, the phase function for fibrous media cannot be specified solely in terms of the scattering angle as in the case of spherical particles. Instead the phase functions depend on the additional parameters which are the incident direction and the fiber orientation. In the consideration of radiative energy transport through fibrous media, the dependence of the phase function on the scattering angle must be expanded in terms of the polar and azimuthal angles ξ and ω of the global coordinate system. The resulting phase function, which is unique for each incident direction, is then used in the equation of transfer.

CONCLUSION

Due to the unique two-dimensional nature of the scattering of radiation by cylindrical fibers, the phase function for a fibrous medium is generally dependent on the directions of both the incident radiation and the fiber orientation. In addition, unlike the phase function for spherical particles whose maximum scattering angle is always π, the maximum scattering angle for fibrous media is dictated by the combination of the incident direction and fiber orientation. For the special case of fibers randomly oriented in space, the phase function is independent of the incident direction and the maximum scattering angle is π.

Although models for diffuse radiative heat transfer through fibers with any orientation have been developed based on the phase function for single fibers [7, 8], their accuracy is limited to the adequacy of the two-flux approximation. As illustrated in the present study, the strong forward peaked phase function for fibrous media implies that the accuracy of the two-flux assumption may be questionable. Detailed knowledge of the phase function allows the evaluation of the transport of both collimated and diffuse radiation through fibrous media by using more accurate solution schemes such as the Monte-Carlo or the discrete ordinate methods.

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FONCTION DE PHASE DE DIFFUSION POUR DES MILIEUX FIBREUX

Résumé—On présente la formulation et l'évaluation de la fonction effective de phase pour des milieux fibreux, à orientation quelconque des fibres, à partir de l'hypothèse que les fibres sont des cylindres infinis et que la diffusion est dans le régime indépendant. La fonction de phase est obtenue en fonction de l'angle de diffusion qui est l'angle entre le rayonnement incident et le rayonnement diffusé. A la différence des particules sphériques dissemblables qui diffusent le rayonnement dans toutes les directions, le rayonnement diffusé par des fibres cylindriques se propage dans des domaines angulaires limités. Par suite, la valeur maximale de l'angle de diffusion pour une collection de fibres dépend de l'orientation des fibres par rapport à la direction d'incidence. Pour une orientation donnée de fibre, la variation angulaire et l'angle maximal de diffusion de la fonction de phase sont différents pour différentes directions incidentes. Si les fibres sont orientées au hasard dans l'espace, la variation angulaire de la fonction de phase est indépendante de la direction incidente et l'angle maximal de diffusion est π. La fonction de phase pour des milieux fibreux montre toujours un pic intense dans la direction du rayonnement incident, ce qui indique que la diffusion est fortement anisotrope.

PHASENFUNKTION DER STREUUNG BEI FASERSTOFFEN

Zusammenfassung—Die Bestimmung der effektiven Phasenfunktion bei Faserstoffen mit beliebiger Faserorientierung wird unter der Annahme beschrieben, daß die Fasern unendlich lange Zylinder sind und die Streuung sich im unabhängigen Bereich befindet. Die Phasenfunktion wird als Funktion des Streuwinkels—dies ist der Winkel zwischen der einfallenden und der gestreuten Strahlung—abgeleitet. Im Gegensatz zu kugeligen Partikeln, welche in alle Richtungen streuen, erfolgt die Streuung bei zylindrischen Fasern nur in einem begrenzten Winkelbereich. Deshalb ist der maximale Streuwinkel bei Streuung an einer Vielzahl von Fasern abhängig vom Winkel zwischen einfallender Strahlung und Faserrichtung. Bei vorgegebener Faserorientierung sind die Verteilung der Streustrahlung und der maximale Streuwinkel der Phasenfunktion bei unterschiedlichen Strahlungseinfallwinkeln ebenfalls unterschiedlich. Bei einer zufälligen räumlichen Anordnung der Fasern ist die Winkelverteilung unabhängig vom Einfallswinkel der Strahlung, und der maximale Streuwinkel ist π. Die Phasenfunktion faserförmiger Stoffe zeigt immer eine deutliche Spitze in Richtung der einfallenden Strahlung. Dies zeigt, daß die Streuung stark anisotrop ist.

ФУНКЦИЯ ФАЗЫ РАССЕЯНИЯ ВОЛОКНИСТЫ СРЕД

Аннотация—Формулируется и оценивается эффективная фазовая функция волокнистых сред с призвольной ориентацией волокон в предположении, что они представляют собой бесконечные цилиндры, а рассеяние осуществляется в независимом режиме. Фазовая функция выражается через угол рассеяния, составленный падающим и рассеянным излучением. В отличие от сферических частиц, рассеивающих излучение во всех направлениях, излучение от цилиндрических волокон распространяется лишь в ограниченных диапазонах угла рассеяния. Следовательно, максимальное значение угла рассеяния в случае излучения от набора волокон зависит от их ориентации относительно направления падающего излучения. При заданной ориентации волокон как зависимость от угла, так и максимальный угол рассеяния фазовой функции различны при разных направлениях падающего излучения. В случае произвольной ориентации волокон в пространстве фазовая функция не зависит от направления падения излучения, а максимальное значение угла рассеяния составляет π. Для фазовой функции волокнистых сред характерен выраженный пик в направлении падающего излучения, что указывает на то, что рассеяние является сильно анизотропным.